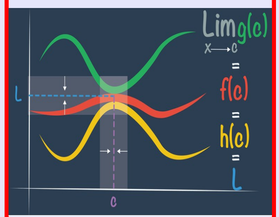
Calculus I

Lecture 9



Feb 19-8:47 AM

Class Quiz 3

Box Your Final Ans.

Given $f(x) = x^2 + 2x + 3$

Find $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 2(x+h) + 3 - x^2 - 2x - 3}{h}$$

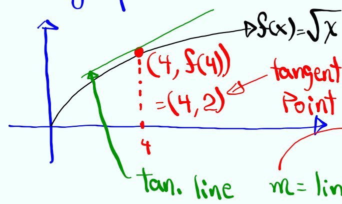
$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{+2x} + 2h \cancel{+3} \cancel{-x^2} \cancel{-2x} \cancel{-3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 2)}{\cancel{h}} = \lim_{h \to 0} (2x + \underbrace{h}_{\to 0} + 2)$$

$$= 2x + 0 + 2 = \boxed{2x + 2}$$

Sep 10-7:05 AM

Find the equation of the tangent line to the graph of $f(x)=\sqrt{x}$ at $x=4$.



$$y-y_1=m(x-x_1)$$

$$y-2=\frac{1}{4}(x-4)$$

$$m=\lim_{h\to 0}\frac{f(4+h)-f(4)}{h}$$

$$m=\lim_{h\to 0}\frac{\sqrt{4+h}-\sqrt{4}}{h}=\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}\quad \frac{0}{0}\text{ I.F.}$$

$$=\lim_{h\to 0}\frac{(\sqrt{4+h}-2)(\sqrt{4+h}+2)}{h(\sqrt{4+h}+2)}=\lim_{h\to 0}\frac{\cancel{4}+\overset{1}{\cancel{h}}-\cancel{4}}{\cancel{h}(\sqrt{4+h}+2)}$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{4+h}+2}=\frac{1}{\sqrt{4+0}+2}=\frac{1}{\sqrt{4}+2}=\boxed{\frac{1}{4}}$$

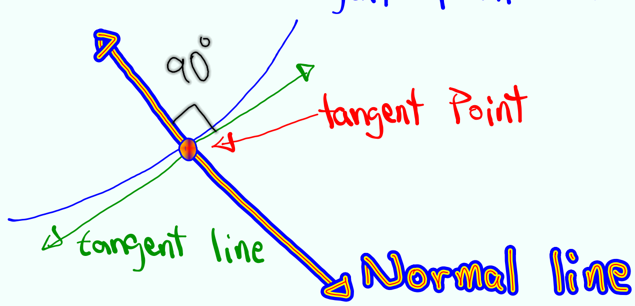
$y-2=\frac{1}{4}(x-4)$ Point- Slope form

$y-2=\frac{1}{4}x-1$

$y=\frac{1}{4}x+1$ Slope-Int. form

Sep 10-7:43 AM

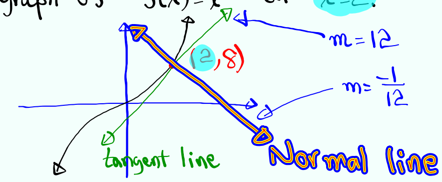
Normal line: It is a line perpendicular to the tangent line at the tangent point on the graph.



$$m_{\text{Normal line}}=\frac{-1}{m_{\text{tangent line}}}$$

Sep 10-7:51 AM

Find equation of the normal line to the graph of $f(x)=x^3$ at $x=2$.



$$m_{tan.\,line} = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$$

Review Binomial thrm

$$= \lim_{h\to 0} \frac{(2+h)^3 - 2^3}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2^3} + 3\cdot 2^2 h + 3\cdot 2 h^2 + h^3 \cancel{-8}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(12+6h+h^2)}{\cancel{h}} = \lim_{h\to 0}(12+6h+h^2) = \boxed{12}$$

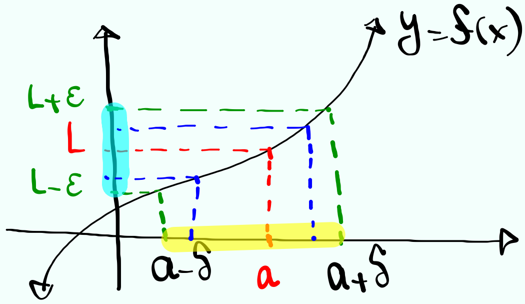
$y - y_1 = m(x - x_1)$

$y - 8 = \frac{-1}{12}(x-2) \Rightarrow \boxed{y=\quad}$

Slope - Int. Form

Sep 10-7:56 AM

Precise definition of limit:



For every $\varepsilon>0$, there is a $\delta>0$ such that $|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$\lim_{x\to a} f(x) = L$ means

For every $\varepsilon>0$, there is a $\delta>0$ Such that

$|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

Sep 10-8:08 AM

Prove $\lim_{x \to 2} (3x+1) = 7$

1) $f(x) = 3x+1$, $L=7$, $a=2$

2) Verify the limit $\lim_{x \to 2} (3x+1) = 3(2)+1 = 7 \checkmark$

3) $|f(x) - L| < \varepsilon$ whenever $|x-a| < \delta$

$|3x+1-7| < \varepsilon$ whenever $|x-2| < \delta$

$|AB| = |A|\,|B|$

$|3x - 6| < \varepsilon$ whenever $|x-2| < \delta$

$|3(x-2)| < \varepsilon$

$|3|\,|x-2| < \varepsilon$

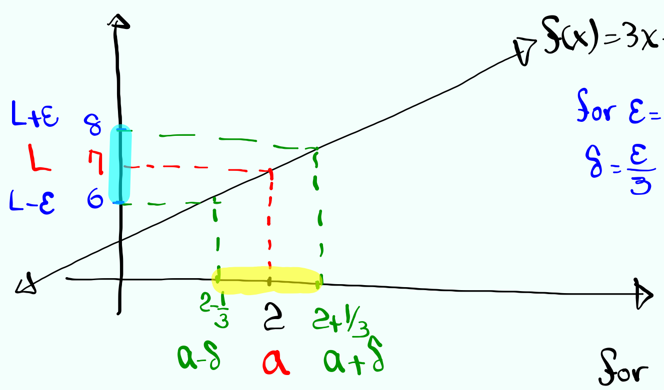
$3|x-2| < \varepsilon$

Divide by 3 $\quad |x-2| < \frac{\varepsilon}{3}$

$\boxed{\text{Pick } \delta = \frac{\varepsilon}{3}}$

$\varepsilon = 1 \rightarrow \delta = \frac{1}{3}$

Sep 10-8:14 AM

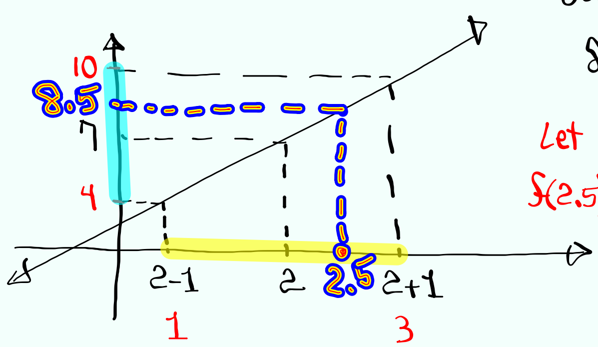


For $\varepsilon = 1$

$\delta = \frac{\varepsilon}{3} = \frac{1}{3}$

For $\varepsilon = 3$

$\delta = \frac{\varepsilon}{3} = \frac{3}{3} = 1$



Let $x = 2.5$

$f(2.5) = 3(2.5)+1$

$= 7.5 + 1$

$= 8.5$

Sep 10-8:21 AM

Prove $\lim_{x \to 4} \left(\frac{1}{4}x + 2\right) = 3$

1) $f(x) = \frac{1}{4}x + 2$, $L = 3$, $a = 4$

2) Verify the limit $\lim_{x \to 4}\left(\frac{1}{4}x + 2\right) = \frac{1}{4}(4) + 2 = 3 \checkmark$

For every $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left|\frac{1}{4}x + 2 - 3\right| < \varepsilon$ whenever $|x - 4| < \delta$

$\left|\frac{1}{4}x - 1\right| < \varepsilon$

Pick $\delta = 4\varepsilon$

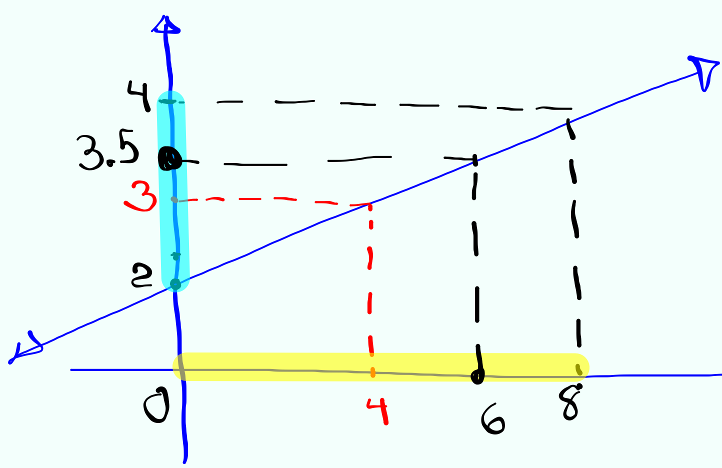
multiply by 4

$|x - 4| < 4\varepsilon$

Let $\varepsilon = 1$

then $\delta = 4$

Sep 10-8:26 AM



$f(x) = \frac{1}{4}x + 2$

$\varepsilon = 1$

$L + \varepsilon = 4$

$L - \varepsilon = 2$

$\delta = 4$

$a + \delta = 8$

$a - \delta = 0$

For $x = 6$ $\quad f(6) = \frac{1}{4}(6) + 2$

$= 1.5 + 2$

$= 3.5$

Sep 10-8:33 AM